

# Package: mhn (via r-universe)

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**Type** Package

**Title** The Modified Half-Normal Distribution

**Version** 0.1.0

**Description** Provides density, distribution, quantile, and random generation functions for the Modified Half-Normal (MHN) distribution, along with moments, mode, and the Fox-Wright Psi function used as the normalizing constant. The MHN distribution arises as a conditional posterior in Bayesian MCMC and generalizes the half-normal, truncated normal, and square-root gamma distributions. Implements efficient sampling via the Sun, Kong & Pal (2023) <doi:10.1080/03610926.2021.1934700> algorithms and the Gao & Wang (2025) <doi:10.1080/03610918.2025.2524551> RTDR method.

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**SystemRequirements** C++17

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**VignetteBuilder** knitr

**URL** <https://github.com/t-momozaki/mhn>,  
<https://t-momozaki.github.io/mhn/>

**BugReports** <https://github.com/t-momozaki/mhn/issues>

**Repository** <https://t-momozaki.r-universe.dev>

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dmhn	<i>Density of the Modified Half-Normal Distribution</i>
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## Description

Computes the probability density function (or log-density) of the Modified Half-Normal (MHN) distribution with parameters alpha, beta, and gamma.

## Usage

```
dmhn(x, alpha = 1, beta = 1, gamma = 0, log = FALSE)
```

## Arguments

x	Numeric vector of evaluation points.
alpha	Shape parameter ( $\alpha > 0$ ). Scalar or numeric vector. Default: 1.
beta	Scale parameter ( $\beta > 0$ ). Scalar or numeric vector. Default: 1.
gamma	Location parameter ( $\gamma \in R$ ). Scalar or numeric vector. Default: 0.
log	Logical; if TRUE, log-density is returned. Default: FALSE.

## Details

The MHN density is

$$f(x \mid \alpha, \beta, \gamma) = \frac{2\beta^{\alpha/2} x^{\alpha-1} \exp(-\beta x^2 + \gamma x)}{\Psi[\alpha/2, \gamma/\sqrt{\beta}]} \quad (x > 0)$$

where  $\Psi[a, z]$  is the Fox-Wright Psi function (Sun et al., 2023, Lemma 1a).

The default parameters  $\alpha = 1$ ,  $\beta = 1$ ,  $\gamma = 0$  correspond to the half-normal distribution  $\text{HN}(1/\sqrt{2})$ .

Special cases are detected and dispatched to closed-form solutions:

- $\gamma = 0$ : sqrt-Gamma distribution
- $\alpha = 1$ : truncated normal distribution

Computation is performed in log-space to avoid numerical underflow/overflow.

When any of  $\alpha$ ,  $\beta$ ,  $\gamma$  is a vector, the density is evaluated element-wise. The Fox-Wright  $\Psi$  normalizing constant is recomputed only when consecutive elements present a different  $(\alpha, \beta, \gamma)$  triple, so passing grouped parameters is significantly faster than calling `dmhn` inside an R loop.

### Value

A numeric vector. The output length equals  $\max(\text{length}(x), \text{length}(\alpha), \text{length}(\beta), \text{length}(\gamma))$ ; each input is recycled to that length following standard R recycling rules. For  $x < 0$ , the density is 0 ( $-\text{Inf}$  if `log = TRUE`).

### References

Sun, J., Kong, M., & Pal, S. (2023). The Modified-Half-Normal distribution: Properties and an efficient sampling scheme. *Communications in Statistics - Theory and Methods*, 52(5), 1507–1536.

### See Also

[mhn\\_mean](#), [mhn\\_var](#), [mhn\\_mode](#)

### Examples

```
x <- seq(0, 5, length.out = 100)
plot(x, dmhn(x, alpha = 2, beta = 1, gamma = 1), type = "l")

# Log-density
dmhn(1, alpha = 2, beta = 1, gamma = 1, log = TRUE)
```

---

mhn\_kurtosis

*Excess Kurtosis of the Modified Half-Normal Distribution*

---

### Description

Computes the excess kurtosis  $\gamma_2 = E[(X - \mu)^4]/\sigma^4 - 3$  for  $X \sim \text{MHN}(\alpha, \beta, \gamma)$ .

### Usage

```
mhn_kurtosis(alpha, beta, gamma)
```

**Arguments**

alpha	Shape parameter ( $\alpha > 0$ ).
beta	Scale parameter ( $\beta > 0$ ).
gamma	Location parameter ( $\gamma \in R$ ).

**Details**

Uses the moment recurrence (Sun et al., 2023, Lemma 2b) to compute raw moments up to fourth order, then converts to central moments.

**Value**

A numeric scalar.

**References**

Sun, J., Kong, M., & Pal, S. (2023). The Modified-Half-Normal distribution: Properties and an efficient sampling scheme. *Communications in Statistics - Theory and Methods*, 52(5), 1507–1536. (Lemma 2b)

**See Also**

[mhn\\_skewness](#), [mhn\\_mean](#)

**Examples**

```
mhn_kurtosis(alpha = 2, beta = 1, gamma = 0)
```

---

mhn\_mean

*Mean of the Modified Half-Normal Distribution*

---

**Description**

Computes  $E(X)$  for  $X \sim \text{MHN}(\alpha, \beta, \gamma)$ .

**Usage**

```
mhn_mean(alpha, beta, gamma)
```

**Arguments**

alpha	Shape parameter ( $\alpha > 0$ ).
beta	Scale parameter ( $\beta > 0$ ).
gamma	Location parameter ( $\gamma \in R$ ).

**Details**

The mean is computed as a ratio of Fox-Wright Psi functions:

$$E(X) = \frac{\Psi[(\alpha + 1)/2, \gamma/\sqrt{\beta}]}{\sqrt{\beta} \Psi[\alpha/2, \gamma/\sqrt{\beta}]}$$

**Value**

A numeric scalar.

**References**

Sun, J., Kong, M., & Pal, S. (2023). The Modified-Half-Normal distribution: Properties and an efficient sampling scheme. *Communications in Statistics - Theory and Methods*, 52(5), 1507–1536. (Lemma 2a)

**See Also**

[mhn\\_var](#), [dmhn](#)

**Examples**

```
mhn_mean(alpha = 2, beta = 1, gamma = 0)
```

---

mhn\_mode

*Mode of the Modified Half-Normal Distribution*

---

**Description**

Computes the mode (most probable value) of the MHN distribution.

**Usage**

```
mhn_mode(alpha, beta, gamma)
```

**Arguments**

alpha	Shape parameter ( $\alpha > 0$ ).
beta	Scale parameter ( $\beta > 0$ ).
gamma	Location parameter ( $\gamma \in R$ ).

**Details**

The mode depends on  $\alpha$ :

$\alpha > 1$   $(\gamma + \sqrt{\gamma^2 + 8\beta(\alpha - 1)})/(4\beta)$  (Sun et al., 2023, Lemma 3b).

$\alpha = 1$   $\max(0, \gamma/(2\beta))$ , obtained as the mode of the truncated normal  $\text{TN}(\gamma/(2\beta), 1/\sqrt{2\beta}, 0, \infty)$  that the MHN reduces to in this case (Sun et al., 2023, Lemma 6b).

$0 < \alpha < 1$  An interior mode exists only when  $\gamma > 0$  and  $\alpha \geq 1 - \gamma^2/(8\beta)$  (Sun et al., 2023, Lemma 3c); otherwise the density is monotonically decreasing (Sun et al., 2023, Lemma 3d) and NA is returned.

**Value**

A numeric scalar. Returns NA when no interior mode exists (density is monotonically decreasing on  $(0, \infty)$ ).

**References**

Sun, J., Kong, M., & Pal, S. (2023). The Modified-Half-Normal distribution: Properties and an efficient sampling scheme. *Communications in Statistics - Theory and Methods*, 52(5), 1507–1536. (Lemma 3b–d, Lemma 6b)

**See Also**

[dmhn](#), [mhn\\_mean](#)

**Examples**

```
mhn_mode(alpha = 2, beta = 1, gamma = 1)
mhn_mode(alpha = 1, beta = 1, gamma = 2)
mhn_mode(alpha = 0.5, beta = 1, gamma = -1) # NA
```

---

mhn\_skewness

*Skewness of the Modified Half-Normal Distribution*

---

**Description**

Computes the skewness  $\gamma_1 = E[(X - \mu)^3]/\sigma^3$  for  $X \sim \text{MHN}(\alpha, \beta, \gamma)$ .

**Usage**

```
mhn_skewness(alpha, beta, gamma)
```

**Arguments**

alpha	Shape parameter ( $\alpha > 0$ ).
beta	Scale parameter ( $\beta > 0$ ).
gamma	Location parameter ( $\gamma \in R$ ).

**Details**

Uses the moment recurrence (Sun et al., 2023, Lemma 2b) to compute raw moments up to third order, then converts to central moments.

**Value**

A numeric scalar.

**References**

Sun, J., Kong, M., & Pal, S. (2023). The Modified-Half-Normal distribution: Properties and an efficient sampling scheme. *Communications in Statistics - Theory and Methods*, 52(5), 1507–1536. (Lemma 2b)

**See Also**

[mhn\\_kurtosis](#), [mhn\\_mean](#)

**Examples**

```
mhn_skewness(alpha = 2, beta = 1, gamma = 0)
```

---

mhn\_var

*Variance of the Modified Half-Normal Distribution*


---

**Description**

Computes  $\text{Var}(X)$  for  $X \sim \text{MHN}(\alpha, \beta, \gamma)$ .

**Usage**

```
mhn_var(alpha, beta, gamma)
```

**Arguments**

alpha	Shape parameter ( $\alpha > 0$ ).
beta	Scale parameter ( $\beta > 0$ ).
gamma	Location parameter ( $\gamma \in R$ ).

**Details**

Uses the formula (Sun et al., 2023, Lemma 2c):

$$\text{Var}(X) = \frac{\alpha}{2\beta} + E(X) \left( \frac{\gamma}{2\beta} - E(X) \right)$$

For  $\alpha \geq 1$ , the variance satisfies  $\text{Var}(X) \leq 1/(2\beta)$  (Sun et al., 2023, Lemma 4c).

**Value**

A numeric scalar.

**References**

Sun, J., Kong, M., & Pal, S. (2023). The Modified-Half-Normal distribution: Properties and an efficient sampling scheme. *Communications in Statistics - Theory and Methods*, 52(5), 1507–1536. (Lemma 2c)

**See Also**

[mhn\\_mean](#), [dmhn](#)

**Examples**

```
mhn_var(alpha = 2, beta = 1, gamma = 0)
```

---

pmhn

*Distribution Function of the Modified Half-Normal Distribution*

---

**Description**

Computes the cumulative distribution function (CDF) of the Modified Half-Normal (MHN) distribution with parameters alpha, beta, and gamma.

**Usage**

```
pmhn(q, alpha = 1, beta = 1, gamma = 0, lower.tail = TRUE, log.p = FALSE)
```

**Arguments**

q	Numeric vector of quantiles.
alpha	Shape parameter ( $\alpha > 0$ ). Scalar or numeric vector. Default: 1.
beta	Scale parameter ( $\beta > 0$ ). Scalar or numeric vector. Default: 1.
gamma	Location parameter ( $\gamma \in R$ ). Scalar or numeric vector. Default: 0.
lower.tail	Logical; if TRUE (default), probabilities are $P(X \leq q)$ , otherwise $P(X > q)$ .
log.p	Logical; if TRUE, probabilities are returned on the log scale. Default: FALSE.

## Details

The CDF is computed via the series representation

$$F(x \mid \alpha, \beta, \gamma) = \frac{1}{\Psi[\alpha/2, \gamma/\sqrt{\beta}]} \sum_{i=0}^{\infty} \frac{z^i}{i!} \Gamma(s_i) P(s_i, \beta x^2)$$

where  $z = \gamma/\sqrt{\beta}$ ,  $s_i = (\alpha + i)/2$ , and  $P(s, y)$  is the regularized lower incomplete gamma function (Sun et al., 2023, Lemma 1b; equivalent to the paper’s form via the identity  $\Gamma(s) P(s, y) = \gamma(s, y)$ , where  $\gamma(s, y)$  is the lower incomplete gamma function used in the paper). The infinite sum is truncated at the constructive bound  $K = \max\{K_1, K_2\}$  from Sun et al. (2023), Supplementary Lemma 10(d), which makes the truncation residual bounded by the user’s tolerance divided by  $\Psi$ . When double-precision cancellation in the alternating-sign accumulator for  $\gamma < 0$  would exceed that tolerance, the series is replaced by a Gauss-Kronrod (or tanh-sinh for  $\alpha < 1$ ) numerical integration of the density on  $[0, q]$ .

Special cases are detected and dispatched to standard R primitives:

- $\gamma = 0$ : `pgamma(q^2, alpha/2, scale = 1/beta)`
- $\alpha = 1$ : truncated-normal CDF via `pnorm`

When any of `alpha`, `beta`, `gamma` is a vector, the CDF is evaluated element-wise. The Fox-Wright  $\Psi$  normalizing constant is recomputed only when consecutive elements present a different  $(\alpha, \beta, \gamma)$  triple, so passing grouped parameters is significantly faster than calling `pmhn` inside an R loop.

## Value

A numeric vector. The output length equals `max(length(q), length(alpha), length(beta), length(gamma))`; each input is recycled to that length following standard R recycling rules. For `q = 0` the CDF is 0; for `q = Inf` it is 1.

## References

Sun, J., Kong, M., & Pal, S. (2023). The Modified-Half-Normal distribution: Properties and an efficient sampling scheme. *Communications in Statistics - Theory and Methods*, 52(5), 1507–1536.

## See Also

[dmhn](#), [qmhn](#), [rmhn](#)

## Examples

```
# Basic evaluation
pmhn(c(0.5, 1, 1.5), alpha = 2, beta = 1, gamma = 1)

# Tail / log forms
pmhn(2, alpha = 2, beta = 1, gamma = 1, lower.tail = FALSE)
pmhn(2, alpha = 2, beta = 1, gamma = 1, log.p = TRUE)

# Special case: gamma = 0 reduces to sqrt-Gamma
all.equal(pmhn(1.5, alpha = 2, beta = 1, gamma = 0),
          pgamma(1.5^2, shape = 1, rate = 1))
```

qmhn

*Quantile Function of the Modified Half-Normal Distribution***Description**

Computes the quantile (inverse cumulative) function of the Modified Half-Normal (MHN) distribution with parameters alpha, beta, and gamma.

**Usage**

```
qmhn(p, alpha = 1, beta = 1, gamma = 0, lower.tail = TRUE, log.p = FALSE)
```

**Arguments**

p	Numeric vector of probabilities.
alpha	Shape parameter ( $\alpha > 0$ ). Scalar or numeric vector. Default: 1.
beta	Scale parameter ( $\beta > 0$ ). Scalar or numeric vector. Default: 1.
gamma	Location parameter ( $\gamma \in R$ ). Scalar or numeric vector. Default: 0.
lower.tail	Logical; if TRUE (default), probabilities are $P(X \leq q)$ , otherwise $P(X > q)$ .
log.p	Logical; if TRUE, probabilities are provided on the log scale. Default: FALSE.

**Details**

For the general case,  $q = F^{-1}(p)$  is obtained by a TOMS 748 root-finder applied to the series CDF (Sun et al., 2023, Lemma 1b). The initial bracket is  $[\sqrt{\epsilon}, E(X) + 8\sqrt{\text{Var}(X)}]$  and is doubled on the right (up to 30 times) until it brackets the target probability.

Special cases are detected and dispatched to standard R primitives:

- $\gamma = 0$ : `sqrt(qgamma(p, alpha/2, scale = 1/beta))`
- $\alpha = 1$ : truncated-normal inverse via `qnorm`

When any of alpha, beta, gamma is a vector, the quantile is evaluated element-wise. The Fox-Wright  $\Psi$  normalizing constant and moments  $E(X)$ ,  $\text{Var}(X)$  (used to size the root-finder bracket) are recomputed only when consecutive elements present a different  $(\alpha, \beta, \gamma)$  triple.

**Value**

A numeric vector. The output length equals `max(length(p), length(alpha), length(beta), length(gamma))`; each input is recycled to that length following standard R recycling rules. `qmhn(0) = 0` and `qmhn(1) = Inf`. Probabilities outside  $[0, 1]$  yield NaN.

**References**

Sun, J., Kong, M., & Pal, S. (2023). The Modified-Half-Normal distribution: Properties and an efficient sampling scheme. *Communications in Statistics - Theory and Methods*, 52(5), 1507–1536.

**See Also**[dmhn](#), [pmhn](#), [rmhn](#)**Examples**

```
# Basic evaluation
qmhn(c(0.1, 0.5, 0.9), alpha = 2, beta = 1, gamma = 1)

# Round-trip: F(F^-1(p)) ~ p
p <- c(0.05, 0.25, 0.5, 0.75, 0.95)
all.equal(pmhn(qmhn(p, alpha = 2, beta = 1, gamma = 1),
              alpha = 2, beta = 1, gamma = 1),
          p, tolerance = 1e-6)

# Tail / log forms
qmhn(0.95, alpha = 2, beta = 1, gamma = 1, lower.tail = FALSE)
qmhn(log(0.05), alpha = 2, beta = 1, gamma = 1, log.p = TRUE)
```

rmhn

*Random Generation from the Modified Half-Normal Distribution***Description**

Draws random variates from the Modified Half-Normal (MHN) distribution with parameters  $\alpha$ ,  $\beta$ , and  $\gamma$ .

**Usage**

```
rmhn(n, alpha = 1, beta = 1, gamma = 0, method = c("auto", "rtdr", "sun"))
```

**Arguments**

<code>n</code>	Non-negative integer giving the number of variates to draw. $n = 0$ returns <code>numeric(0)</code> .
<code>alpha</code>	Shape parameter ( $\alpha > 0$ ). Scalar or numeric vector. Default: 1.
<code>beta</code>	Scale parameter ( $\beta > 0$ ). Scalar or numeric vector. Default: 1.
<code>gamma</code>	Location parameter ( $\gamma \in R$ ). Scalar or numeric vector. Default: 0.
<code>method</code>	Sampling algorithm. One of "auto" (default), "rtdr", or "sun". See Details.

**Details**

The MHN density is

$$f(x \mid \alpha, \beta, \gamma) = \frac{2\beta^{\alpha/2} x^{\alpha-1} \exp(-\beta x^2 + \gamma x)}{\Psi[\alpha/2, \gamma/\sqrt{\beta}]} \quad (x > 0)$$

where  $\Psi[a, z]$  is the Fox-Wright Psi function. `rmhn` does not evaluate  $\Psi$ ; the rejection-sampling kernels cancel it out.

The default parameters  $\alpha = 1$ ,  $\beta = 1$ ,  $\gamma = 0$  correspond to the half-normal distribution  $\text{HN}(1/\sqrt{2})$ .

The method argument selects the rejection sampler:

- "auto": Special-case shortcuts when applicable ( $\gamma \approx 0 \rightarrow$  sqrt-Gamma,  $\alpha \approx 1 \rightarrow$  truncated normal). Otherwise dispatches to RTDR (Gao & Wang, 2025).
- "rtdr": Force the Relaxed Transformed Density Rejection method of Gao & Wang (2025). The acceptance probability is bounded below by  $1/e \approx 0.368$  uniformly over the parameter space. Note: Gao & Wang (2025) use the parameterization  $(\lambda, \alpha, \beta)$  with density proportional to  $x^{\lambda-1} \exp(-\alpha x^2 - \beta x)$ ; the mapping to the Sun et al. parameterization used here is  $\lambda \leftrightarrow \alpha$ ,  $\alpha \leftrightarrow \beta$ ,  $\beta \leftrightarrow -\gamma$  (sign flip on the linear term).
- "sun": Force the Sun et al. (2023) algorithms. Algorithm 1 is used when  $\gamma > 0$  and  $\alpha > 1$ ; Algorithm 3 is used when  $\gamma \leq 0$ . The combination  $\alpha < 1$  with  $\gamma > 0$  is unsupported and triggers an error.

Vector parameters are recycled to length  $n$  following standard R rules. Trailing parameter elements beyond index  $n - 1$  are silently ignored, matching the convention of `rnorm`.

Internally the setup state of the chosen sampler is reused as long as consecutive  $(\alpha, \beta, \gamma)$  triples are equal, so passing parameters grouped by triple is faster than calling `rmhn` inside an R loop.

### Value

A numeric vector of length  $n$ . If any of `alpha`, `beta`, `gamma` (after recycling to length  $n$ ) is NA or non-finite (Inf, -Inf, NaN), the corresponding output element is NA.

### References

- Sun, J., Kong, M., & Pal, S. (2023). The Modified-Half-Normal distribution: Properties and an efficient sampling scheme. *Communications in Statistics - Theory and Methods*, 52(5), 1507–1536.
- Gao, F. & Wang, H.-B. (2025). Generating modified-half-normal random variates by a relaxed transformed density rejection method. *Communications in Statistics - Simulation and Computation*.
- Robert, C. P. (1995). Simulation of truncated normal variables. *Statistics and Computing*, 5(2), 121–125.

### See Also

[dmhn](#), [mhn\\_mean](#), [mhn\\_var](#)

### Examples

```
set.seed(1)
rmhn(10, alpha = 2, beta = 1, gamma = 0.5)

# Vector parameters are recycled to length n.
set.seed(1)
rmhn(5, alpha = c(1, 2, 3, 4, 5))
```

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